EVIDENCE FOR INERTIAL OSCILLATIONS ALONG TRANSOSONDE TRAJECTORIES

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ABSTRACT

Carefully screened transosonde flights are analyzed for evidences of inertial oscillations. The most frequent period of wind speed oscillation along these flights is found to be 12 hr., considerably shorter than the theoretical mode for inertial oscillations. However, when the data are subdivided according to latitude, there is a tendency for wind speed oscillations appropriate to the theoretical inertial period to occur with above average frequency. This tendency is less pronounced when the period of wind speed oscillation is compared with the curvature of the geostrophic flow and is scarcely noticeable when the period of oscillation is compared with the horizontal wind shear. Combining these effects, it is shown that, to a good approximation, wind speed oscillations of above average frequency vary with the absolute (geostrophic) vorticity along the trajectories in the manner theoretically prescribed for inertial oscillations. There is also a tendency for an above average frequency of very short period wind oscillations when the anticyclonic angular velocity of the flow is large. It is tentatively suggested that this phenomenon is associated with the occurrence of "abnormal" flow.

1. INTRODUCTION

During the period 1957–59 the United States Navy launched constant level balloons (transosondes) from Iwakuni, Japan at the rate of about one flight every two days. The balloons were set to float at 300 or 250 mb. with the flight duration pre-set at 4 to 5 days. Balloon positioning was accomplished at 2-hr. intervals by means of intercepts obtained from the radio direction-finding networks of the United States Navy and the Federal Communications Commission (FCC). For details of transosonde operation in general, and these operational flights in particular, see articles by Anderson and Mastenbrook [1] and Angell [2].

The present article deals solely with the evidence for inertial oscillations to be obtained from these operational flights from Japan. Inertial oscillations are those oscillations which an air parcel, disturbed from geostrophic equilibrium, undergoes if there are no forces acting on the parcel other than pressure gradient and Coriolis forces.

If the geostrophic wind field is constant in space and time, the inertial period is given by $2\pi/f$ where f is the Coriolis parameter. Under these special conditions the inertial period increases with decreasing latitude, possessing a value of about 18 hr. at latitude 40°. If the geostrophic wind field is not constant in space and time the theoretical determination of the inertial period is not so simple. With the assumption that within a small area the geostrophic wind field may be approximated by a stream function quadratic in x, y, and t (horizontal space coordinates and time), Perkins [3] has shown that, disregarding the convergence or divergence of contours, the inertial period equals $(f+\zeta_g)^{-1/2}$ half pendulum-days where f and ζ_g (the relative geostrophic vorticity) are evaluated in time units of half pendulum-days divided by 2π (i.e., $f=1$). Thus the inertial period is increased if the geostrophic wind shear is anticyclonic and there is anticyclonic curvature of isobars or contours. In the extreme case, as the absolute geostrophic vorticity approaches zero, the inertial period approaches infinity and inertial instability is said to exist. In this paper the variation of transosonde-derived wind speed periodicities with respect to the Coriolis parameter (latitude) and relative geostrophic vorticity (horizontal shear and angular velocity) is compared with the theoretical variation of the inertial period with respect to these parameters as determined from Perkins' equation.

2. PROCEDURES

Owing to the relatively small magnitude of inertial oscillations, it is necessary to restrict the analysis to the more accurately positioned segments of transosonde trajectories. Accordingly, only portions of trajectories east of the 180th meridian which were accurately positioned by the FCC for at least 36 hr. were utilized in this investigation. Even with this screening some smoothing of the data appeared desirable. The smoothing consisted of a one-two-one weighting of each 2-hourly latitude and longitude. The velocity was then determined from the distance and direction between successive 2-hourly smoothed positions. For this particular analysis, only the wind speed is investigated from the viewpoint of inertial oscillations.

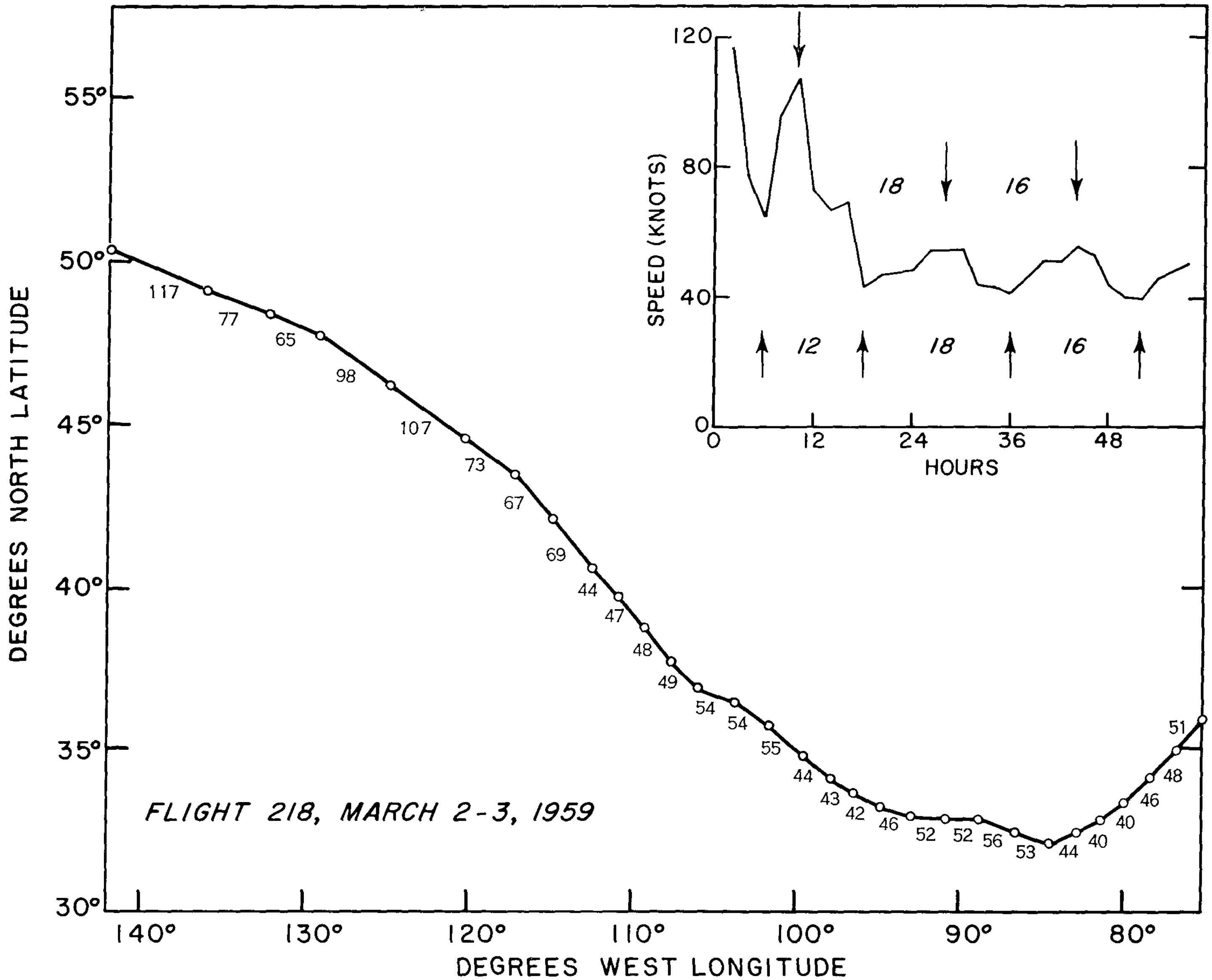

FIGURE 1.—Portion of the trajectory of transosonde flight 218 with 2-hourly smoothed positions and speeds in knots indicated along the trajectory. The inset shows the time trace of the speeds and indicates the number of hours between speed maxima and speed minima. The latitude scale is slightly exaggerated in order to emphasize the sinuosities in the trajectory.

From the traces of transosonde-derived wind speed as a function of time, it is necessary to set up a criterion as to what constitutes a true periodicity in wind speed and what constitutes a fictitious periodicity due to errors in transosonde positioning. The criterion established, necessarily subjective, was that wind speed changes exceeding 10 kt. were real and represented a true periodicity in the flow whereas speed changes of less than this amount were fictitious and were due to errors in balloon positioning. Estimates of the errors in transosonde positioning suggest that 10 kt. is a reasonable dividing line from this point of view and yet one would anticipate that most oscillations of inertial character would be of sufficient magnitude so as not to be eliminated by such a dividing line. With this criterion, more than 500 wind speed periodicities were determined from the transosonde trajectories, based on the number of hours between wind speed maxima and the number of hours between wind speed minima.

3. AN EXAMPLE OF INERTIAL OSCILLATIONS

As an example of the procedure applied, and of oscillations of inertial character, figure 1 shows a portion of the trajectory of transosonde flight 218 with 2-hourly positions smoothed in the manner indicated above and with the derived wind speed in knots indicated between positions. The inset shows a time trace of these speeds with the speed maxima and minima delineated by arrows and the number of hours between successive maxima and minima indicated by the numerals. Note that with one exception the number of hours between speed maxima and between speed minima corresponds well to the theoretical inertial period for the given latitude and furthermore that the speed maxima occur near the crests of the small, wave-like oscillations along the trajectory, as would be expected of inertial oscillations. The best evidence for inertial oscillations along individual trajectories has been found with such a trajectory configuration, namely, when a long fetch exists between crest and downstream trough line.

4. STATISTICAL EVIDENCE FOR INERTIAL OSCILLATIONS

Let us now look at the statistical evidence for inertial oscillations. The italicized numbers along the bottom of figure 2 indicate the percentage of wind speed oscillations of given period along the transosonde trajectories. Thus, 12.8 percent of the more than 500 wind speed oscillations were of 12-hr. period (modal value) whereas only 9.3 percent were of 18-hr. period. Certainly the frequency histogram is peaked at a lower period than would theoretically be expected of pure inertial oscillations. It is difficult to state how significant this deviation is because, on the one hand, a bias toward relatively high-frequency oscillations is introduced by errors in transosonde positioning but, on the other hand, very-high-frequency oscillations are eliminated by the smoothing of the positions. Faced with the fact that the modal period of wind speed oscillation derived from the transosonde data does not coincide with the period to be expected of inertial oscillations, let us investigate the manner in which the frequency of wind speed oscillation at a given period varies with latitude, curvature of the geostrophic flow, and horizontal shear of the geostrophic wind, i.e., those parameters which induce a variation in the inertial period.

LATITUDE EFFECT

If inertial oscillations are present, then within any given latitude band there should be a greater percentage of wind speed oscillations of period equal to the inertial period than at other latitudes where the inertial period would be different. In order to utilize this reasoning, for each latitude band the percentage of wind speed oscillations of given period was determined and then, for each period, the deviation of this percentage from the mean value for all latitudes was evaluated. Thus in figure 2 the number -3.2 at a period of 14 hr. and a latitude of 30° means that $12.1-3.2$ or 8.9 percent of the wind speed oscillations at latitude 30° were of 14-hr. period. On the other hand at latitude 50° $12.1+4.6$ or 16.7 percent of the wind speed oscillations were of 14-hr. period. On this basis, the stippled areas in figure 2 indicate those periods and latitudes which possess an above average percentage of wind speed oscillations. It is immediately apparent that the trend of the main stippled area agrees well with the theoretical variation with latitude of the inertial period, as given by the dotted line in figure 2. Thus there is fairly convincing evidence for an above average per-

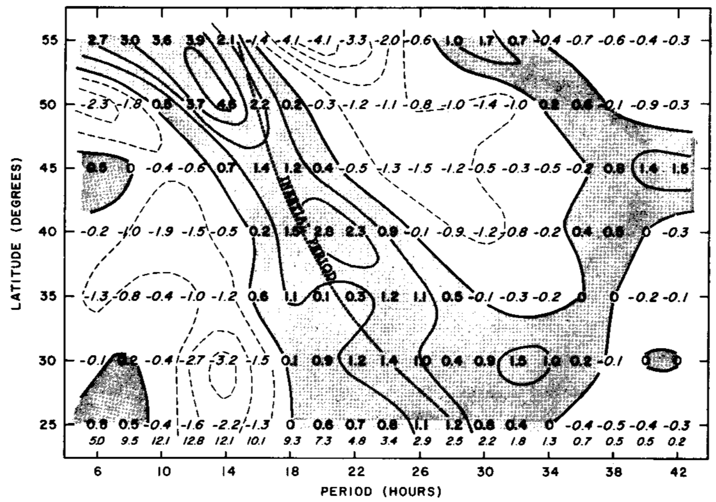

FIGURE 2.—Transosonde-derived wind speed periodicity as a function of latitude. The italicized numbers at the bottom of the diagram give the percentage of oscillations of given period for all latitudes. The numbers in the body of the diagram give for each period, the deviation at each latitude from this mean percentage. The stippled area shows where the wind speed periodicities are of above average frequency. The dotted line indicates the theoretical variation of inertial period with latitude.

centage of wind speed oscillations with the appropriate inertial period even though the modal value of the periodicity does not correspond to the inertial period. In passing it might also be suggested that the stippled area in figure 2, trending in the same manner as the above but at periods of oscillation exceeding 30 hr., may reflect the passage of the transosondes through the long waves in the westerlies, which at lower latitudes result in a well-marked oscillation in the meridional wind of period near 48 hr. [4].

CURVATURE EFFECT

The inertial period is also a function of the angular velocity of the geostrophic flow, that is, the product of geostrophic wind speed and curvature of isobars or contours. For simplicity and objectivity, however, this product was approximated by $d\theta/dt=VK$ where $d\theta/dt$ is the readily available angular velocity of the transosonde, V is wind speed, and K is trajectory curvature. For each wind speed period a value of $d\theta/dt$ was determined from the change in direction over the period interval and subsequently figure 3 was obtained, similar to figure 2 except that the ordinate is now angular velocity rather than latitude. The numbers in the body of figure 3 have been omitted for clarity and simplicity. Before discussing figure 3 it should be noted that inasmuch as ridges occur at more northerly latitudes than troughs, the curvature influence on the inertial period is usually counterbalanced to some degree by the latitude influence so that the expected relationship between curvature and inertial period could be strongly masked. The most obvious feature about figure 3 is the symmetry of the stippled area such

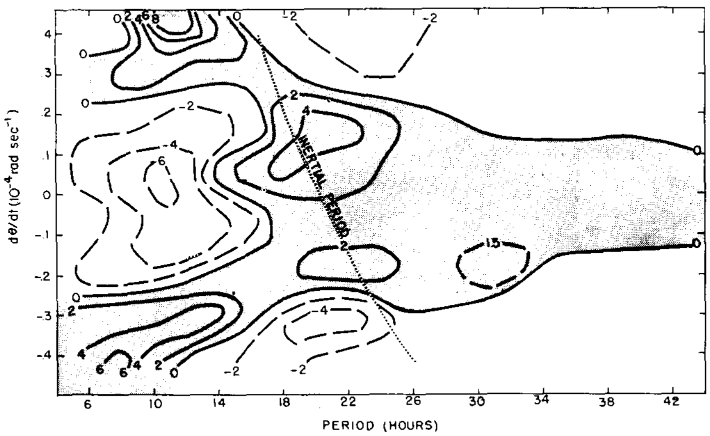

Figure 3.—Transosonde-derived wind speed periodicity as a function of transosonde angular velocity. The stippled area shows where the wind speed periodicities are of above average frequency while the dotted line indicates the theoretical variation of inertial period with (geostrophic) angular velocity.

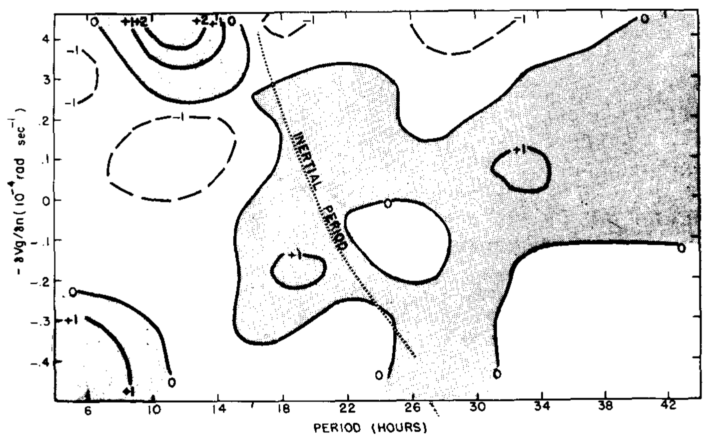

Figure 5.—Transosonde-derived wind speed periodicity as a function of horizontal geostrophic wind shear (n directed to the left of the trajectory looking downstream). The stippled area shows where the wind speed periodicities are of above average frequency while the dotted line indicates the theoretical variation of inertial period with horizontal geostrophic wind shear.

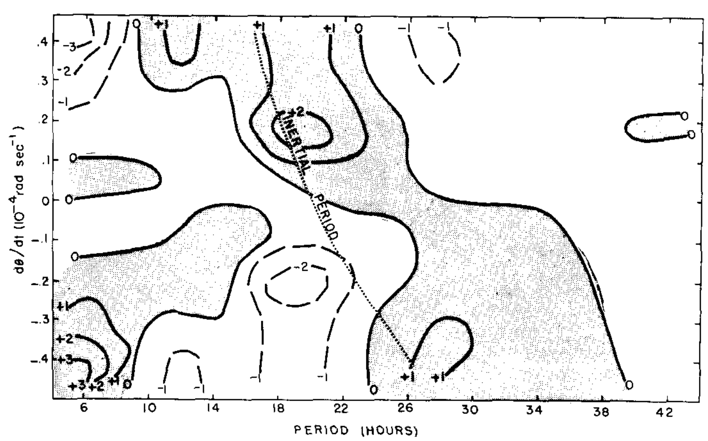

Figure 4.—Transosonde-derived wind speed periodicity as a function of transosonde angular velocity, expressed as a deviation from symmetry about the zero ordinate value in figure 3. The stippled areas show where the wind speed periodicities are of above average frequency while the dotted line indicates the theoretical variation of inertial period with (geostrophic) angular velocity.

that short-period oscillations are more frequent than average both when the flow is cyclonically curved and when it is anticyclonically curved. Hence the looked-for association between geostrophic curvature and theoretical inertial period (dotted line) is not immediately obvious. However, upon determining the deviation from symmetry about the zero ordinate in figure 3 it is found that fair agreement exists between the variation with angular velocity of the theoretical inertial period and the variation with angular velocity of wind speed oscillations of above average frequency, as shown by the main stippled area in figure 4. Still to be explained in figure 4, however, is the large percentage of wind speed oscillations of very short period when the angular velocity has a large anticyclonic (negative) value. It is seen from the ordinate values that these short-period oscillations occur when the flow is on the verge of being "abnormal" or anomalous, that is, on the verge of possessing anticyclonic rotation in space. It may be that under such extreme conditions a different type of stability and periodicity is to be expected.

SHEAR EFFECT

The last feature to be examined is the variation of wind speed period with horizontal (geostrophic) wind shear. Since it is impossible to obtain the horizontal wind shear from a single transosonde trajectory, it is necessary to utilize analyzed maps to obtain values of this quantity. Consequently, for the first time we are forced to use data not derived directly from the transosondes themselves, thus introducing a subjective element in the analysis. The maps used to obtain values of the horizontal geostrophic wind shear along the trajectories were National Weather Analysis Center maps for 300 and 250 mb. The shear was evaluated at 12-hr. intervals at the position of the transosonde at map time (using a 5° grid) and then linear interpolation was applied to estimate the mean value of the shear over the given wind speed period. Figure 5 shows the percentage deviation of wind speed oscillations as a function of period and horizontal geostrophic shear. Obviously, there is little association between theoretical inertial period and above average frequency of wind speed periodicity. Interestingly enough, however, there is evidence of frequent short-period oscillations in wind speed both when the shear is strongly cyclonic and strongly anticyclonic, in agreement with the results presented in figure 3. Since shear and angular velocity (or curvature) would not be expected to be related, this finding tends to confirm the observation

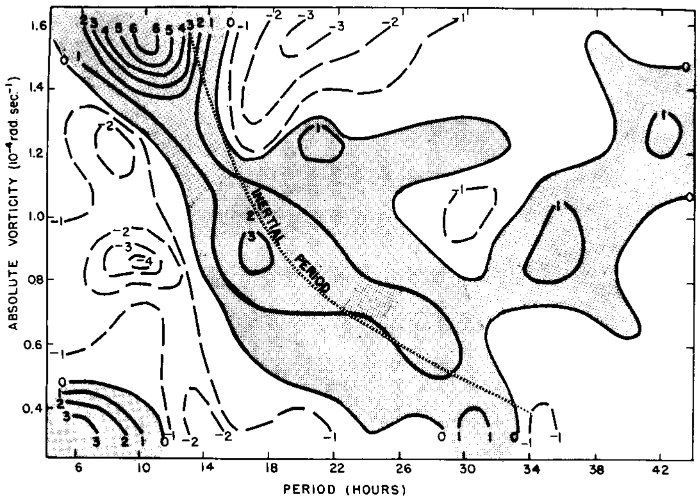

FIGURE 6.—Transosonde-derived wind speed periodicity as a function of absolute (geostrophic) vorticity along the trajectory. The stippled area shows where the wind speed periodicities are of above average frequency while the dotted line indicates the theoretical variation of inertial period with absolute (geostrophic) vorticity.

that wind speed oscillations along a trajectory are of shorter period when the absolute magnitude of the vorticity along the trajectory is large.

COMBINED EFFECT

It was shown in the introduction that with certain approximations the theoretical inertial period is a function of the absolute geostrophic vorticity along the trajectory. Therefore, the inertial periodicity should show up most clearly in the wind speed oscillations if the above effects of earth vorticity (function of latitude) and geostrophic vorticity due to shear and curvature are combined. Figure 6 shows the results obtained by carrying out such a combination of parameters. In view of the clear-cut influence of the latitude upon the inertial period (fig. 2) it is not surprising that in figure 6 as well there is good agreement between theoretical inertial period as a function of absolute (geostrophic) vorticity (dotted line) and the stippled area representing regions of above average frequency of wind speed oscillations. This agreement diminishes at relatively small values of the absolute vorticity, however, and, as stated previously, at these small values wind speed oscillations of very short period tend to predominate. Further discussion of this latter finding will be reserved for a subsequent paper.

5. CONCLUSION

It has been shown that, using carefully screened transosonde data, good evidence for the existence of inertial oscillations can be obtained, both along individual flights and statistically utilizing the data from all flights. Using these same carefully screened data, other analyses are planned dealing with phenomena such as inertial instability, "abnormal" flow, and the possible forecast implications of ageostrophic flow at jet stream level. It is hoped thereby to reap all possible benefits from the unique data obtainable from this series of operational transosonde flights.

ACKNOWLEDGMENTS

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